UK Maths Trust

Grey Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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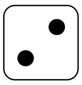
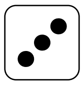
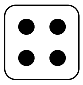
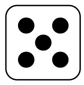
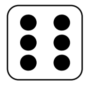
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
E	E	C	B	B	B	A	B	D	D	C	C	D	E	A	B	B	D	E	C	C	D	A	D	A

1. Sandra rolls three dice and gets a total of 8. The three dice show different numbers of dots.
Which number of dots could Sandra *not* have rolled on any of her dice?

A  B  C  D  E 

SOLUTION **E**

It is possible for Sandra to obtain a total of 8 from rolling three dice and getting three different numbers if she rolls 1, 2 and 5 or 1, 3 and 4. Hence Sandra could have rolled 1, 2, 3, 4 and 5. However, if Sandra rolls a 6, then the other two numbers needed to get a total of 8 would both need to be 1 and hence this combination would not be made up of three different numbers. Therefore, the number of dots Sandra could not have rolled is 6.

2. Daniel is 5 years old. His brother Dominic is 6 years older.
What will the sum of their ages be in 7 years' time?

A 26 B 27 C 28 D 29 E 30

SOLUTION **E**

In seven years' time, Daniel's age will be $5 + 7 = 12$. In seven years' time, Dominic's age will be $5 + 6 + 7 = 18$. Therefore, in seven years' time, the sum of their ages will be $12 + 18 = 30$.

3. Ohad writes the four digits 2, 0, 2 and 5, in some order, in the boxes shown. He then does the sum he has created.

$$\square - \square + \square - \square$$

What is the smallest value that Ohad could obtain?

A -7 B -6 C -5 D -4 E -3

SOLUTION **C**

To obtain the smallest value, Ohad would place the two largest digits after the subtraction signs. These digits are 5 and one of the 2s. Hence, one possible calculation that gives the smallest possible value is $2 - 2 + 0 - 5$. This gives the smallest value he could obtain as -5.

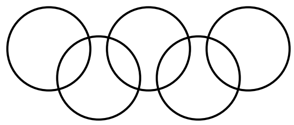
4. There were ten more truth-tellers than liars in a room. Everyone in the room was asked, “Are you a truth-teller?” and everyone gave an answer. A total of 20 people answered, “Yes.”
How many liars were in the room?

A 0 B 5 C 15 D 20 E 25

Solution **B**

Both a truth-teller and a liar will answer "Yes" to the question "Are you a truth-teller?". Since 20 people answered "Yes", there is a total of 20 people in the room. The question tells us that there are 10 more truth-tellers than liars. Hence, if the numbers of truth-tellers and liars are represented by T and L respectively, we have $T + L = 20$ and $T - L = 10$. Therefore $2L = 20 - 10$ and hence the number of liars in the room is $\frac{20 - 10}{2} = 5$.

5. Five circles, each with an area of $8\,\text{cm}^2$, overlap each other to form the figure shown. The area of each section where two circles overlap is $1\,\text{cm}^2$.
What is the total area covered by the figure?



A $32\,\text{cm}^2$ B $36\,\text{cm}^2$ C $39\,\text{cm}^2$ D $41\,\text{cm}^2$ E $42\,\text{cm}^2$

Solution **B**

There are four regions where two circles overlap. Therefore, the total area covered by the overlaps is $4\,\text{cm}^2$. The total area of the five circles, were they not to overlap, would be $5 \times 8\,\text{cm}^2 = 40\,\text{cm}^2$. Hence the total area covered by the figure is $(40 - 4)\,\text{cm}^2 = 36\,\text{cm}^2$.

6. Ria has some £1 coins and some £2 coins in her pocket. She has 50% more £1 coins than £2 coins. She has £35 in total. How many £2 coins does Ria have?

A 12 B 10 C 8 D 6 E 4

Solution **B**

Let the number of £2 coins Ria has be $2X$. Therefore she has $3X$ £1 coins. The total value of her coins, in pounds, is $1 \times 3X + 2 \times 2X = 7X$. We are told that she has £35 in total and hence $7X = 35$, which has solution $X = 5$. Therefore the number of £2 coins Ria has is $2 \times 5 = 10$.

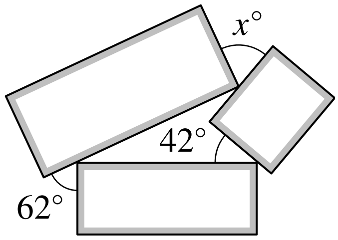
7. There are five hurdles in a 60 m hurdles race. The first hurdle is 12 m from the start. The gap between any two consecutive hurdles is 8 m.
How far is the last hurdle from the finish?

A 16 m B 14 m C 12 m D 10 m E 8 m

Solution **A**

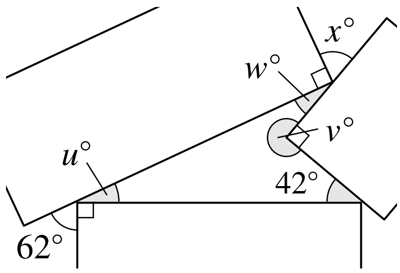
Since the distance to the first hurdle is 12 m and the distance between each of the five hurdles is 8 m, the distance from the final hurdle to the finish is $(60 - 12 - 4 \times 8)$ m $= 16$ m.

8. Louisa places three rectangular pictures in the way shown.
What is the value of x?



A 64 B 70 C 72 D 76 E 80

Solution **B**

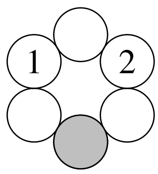


Let the unknown angles in the quadrilateral formed between the three pictures be $u°$, $v°$ and $w°$, as shown in the diagram.

Since angles at a point add to 360°, we have $90 + v = 360$ and hence $v = 270$. Since angles on a straight line add to 180°, we have $62 + 90 + u = 180$ and hence $u = 28$. Since angles in a quadrilateral add to 360°, we have $u + 42 + v + w = 360$ and hence $w = 20$.

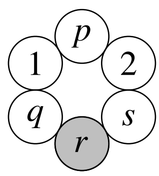
Finally, using angles on a straight line again, we have $w + 90 + x = 180$ and hence $x = 70$.

9. Eddie wants to write a number in each circle in the diagram. He wants each number to be equal to the sum of the numbers in the two adjacent circles. He has already written two numbers, as shown. What number should he write in the grey circle?



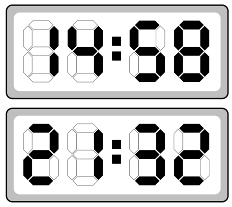
A 2 B −1 C −2 D −3 E −4

Solution **D**



Let the numbers in the empty circles be p, q, r and s, as shown in the diagram. Since the number in each circle is equal to the sum of the numbers in the two circles on either side of it, we know that $p = 1 + 2 = 3$. Then, since $p + q = 1$ and $p + s = 2$, we have $q = -2$ and $s = -1$. Therefore, since $r = q + s$, the number Eddie should write in the shaded circle is $-2 + (-1) = -3$.

10. Werner is on a treadmill in the gym. He keeps looking at two stopwatches. The first shows the time elapsed since he started and the second the time remaining until the end of his session. The diagram shows the times displayed by these two stopwatches at some point during the session.
What time do the stopwatches show when they display the same times?



A 17:50 B 18:00 C 18:12 D 18:15 E 18:20

Solution **D**

Since one stopwatch is running forward and one is running backwards, the total time of Werner's session is given by the sum of the times at any given instant. Therefore the total time of his session is 21 minutes 32 seconds + 14 minutes 58 seconds = 36 minutes 30 seconds. The two stopwatches will show the same time halfway through the session when both stopwatches show half of 36 minutes 30 seconds, which is 18 minutes 15 seconds. Hence the time shown will be 18 : 15.

11. Julia wants to fill in each $\square$ with a different prime number less than 20 so that the value of X is an integer. What is the maximum possible value of X?

$$X = \frac{\square + \square + \square + \square + \square + \square + \square}{\square}$$

A 20 B 14 C 10 D 8 E 6

Solution **C**

The eight prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19, which have sum 77. Let the prime number that is placed in the denominator of the calculation of X be Y. For the value of X to be an integer, $77 - Y$ is divisible by Y and hence 77 is divisible by Y. Since $77 = 7 \times 11$, the only possible values for Y are 7 and 11 and, since $\frac{(77 - Y)}{Y} = \frac{77}{Y} - 1$, the maximum value of X will occur when Y is as small as possible, and hence occurs when Y is 7. This maximum value is $\frac{77 - 7}{7} = 10$.

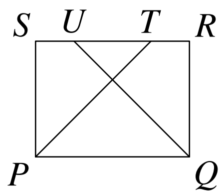
12. The integers a, b, c and d satisfy $a < 2b$, $b < 3c$, $c < 4d$, and $d < 100$. What is the largest possible value of a?

A 1200 B 2000 C 2367 D 2399 E 2400

SOLUTION **C**

Since $d < 100$ we have $d \leq 99$. Then, since $c < 4d$, we have $c < 4 \times 99 = 396$ and hence $c \leq 395$. Next, since $b < 3c$, we have $b < 3 \times 395 = 1185$ and hence $b \leq 1184$. Finally, since $a < 2b$, we have $a < 2 \times 1184 = 2368$ and hence $a \leq 2367$. Therefore the largest possible value of a is 2367.

13. In the rectangle $PQRS$, the points T and U are marked on side SR as shown, so that $\angle RUQ = \angle STP = 45°$ and $PQ + UT = 20$ cm. What is the length of QR?



A 4 cm B 6 cm C 8 cm D 10 cm E 12 cm

SOLUTION **D**

Since $PQRS$ is a rectangle, $\angle PST = \angle QRU = 90°$. Also, since angles in a triangle add to 180° and we are told that $\angle RUQ = \angle STP = 45°$, we have $\angle RQU = \angle SPT = 45°$. Hence both triangles TSP and URQ are isosceles. Therefore $QR = RU$ and $PS = ST$. Let the lengths of PS, ST, RU and QR, which are all equal, be y cm and let the length of SR be x cm. From the diagram, we can see that $PQ + UT = SR + UT$ and this is equal to $(x + (2y - x))$ cm. This simplifies to $2y$ cm and hence $2y = 20$. This has solution $y = 10$ and therefore the length of QR is 10 cm.

14. Sanja has two bowls of numbered balls. Bowl X contains seven balls numbered 1, 2, 6, 7, 10, 11 and 12. Bowl Y contains five balls numbered 3, 4, 5, 8, and 9. Which ball should Sanja transfer from Bowl X to Bowl Y to increase the mean of the numbers on the balls in each bowl?

A 12 B 11 C 10 D 7 E 6

SOLUTION **E**

The total of the numbers on the seven balls in Bowl X is 49 and hence the mean is $49 \div 7 = 7$. The total of the numbers on the five balls in bowl Y is 29 and hence the mean is $29 \div 5 = 5.8$. To increase the mean of the numbers on the balls in bowl X, the number on the ball removed will be smaller than 7. To increase the mean of the numbers on the balls in bowl Y, the number on the ball added will be larger than 5.8. The only number on a ball in bowl X that satisfies both of these conditions is 6. Therefore the number on the ball that Sanja should move is 6.

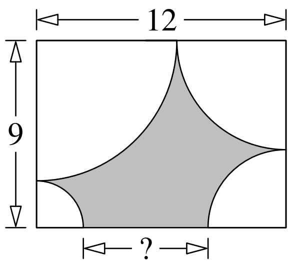
15. In the six-digit integer $PAPAYA$, different letters stand for different digits and the same letter always represents the same digit. Also $Y = P + P = A + A + A$.
What is the value of $P \times A \times P \times A \times Y \times A$?

A 432 B 342 C 324 D 243 E 234

Solution **A**

Since we are told that $Y = P + P = A + A + A$, we know that Y is both a multiple of 2 and a multiple of 3. The only such digit is 6 and hence $Y = 6$, $P = 3$ and $A = 2$. Therefore the value of $P \times A \times P \times A \times Y \times A$ is $3 \times 2 \times 3 \times 2 \times 6 \times 2 = 6^3 \times 2 = 432$.

16. Peter has drawn a rectangular flag with dimensions 12 cm by 9 cm. He has drawn a quarter-circle with centre at each corner and coloured the region formed, as shown.
What is the length indicated by the question mark?

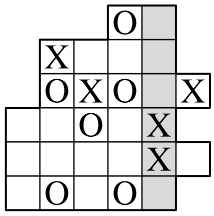


A 5 cm B 6 cm C 7 cm D 8 cm E 9 cm

Solution **B**

Consider the vertical sides of the flag. The sum of the radii of the four quarter-circles is equal to the sum of the lengths of these vertical sides and so is $(9 + 9)$ cm = 18 cm. The sum of the radii of the two largest quarter-circles is equal to the length of the horizontal side of the flag and so is 12 cm. Therefore the sum of the radii of the two smallest quarter-circles is equal to $(18 - 12)$ cm = 6 cm. Therefore the length indicated with a question mark on the diagram is $(12 - 6)$ cm = 6 cm.

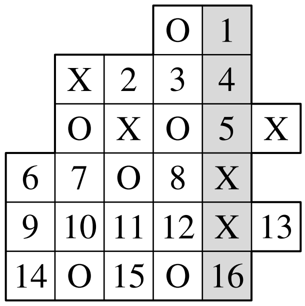
17. Morten fills in the cells on the diagram shown so that each cell contains either an X or an O. He also ensures there is no line of four consecutive identical symbols in any column, row or diagonal. When he has completed the diagram, what will the column coloured grey contain?

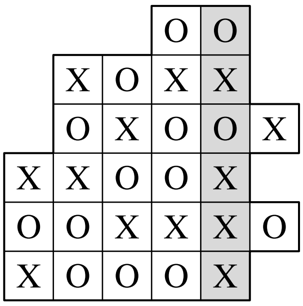


A 3 Os and 3 Xs B 2 Os and 4 Xs C 4 Os and 2 Xs
D 5 Os and 1 X E 1 O and 5 Xs

Solution **B**

Number the empty cells in the diagram 1 to 16, as shown. In the description that follows, each cell is filled to avoid a line of four consecutive identical symbols. First place an O in cell 8. Then place an X in both cells 3 and 12. Next place an O in cell 15. Then place an X in both cells 14 and 16. Next place an O in cell 5 and then an X in cell 11. Then place an O in both cells 10 and 13 and then an X in both cells 7 and 4. Next place an O in all of cells 1, 2 and 9 and finally place an X in cell 6. The completed diagram is as shown in the second diagram, with the grey coloured column containing 2 Os and 4 Xs.





18. During two sessions of football training, Paul shoots a total of 17 times at a target. He hits with 60% of his shots in the first session. He hits with 75% of his shots in the second session.
How many times did he hit the target in the second session?

A 6 B 7 C 8 D 9 E 10

Solution **D**

Since Paul hits with $60\% = \frac{3}{5}$, of his shots in his first session, the number of shots he takes in the first session is a multiple of 5. Similarly, since Paul hits with $75\% = \frac{3}{4}$, of his shots in his second session, the number of shots he takes in the second session is a multiple of 4. We are told that Paul takes 17 shots in total and hence these must be split into a multiple of 5 and a multiple of 4. The only possible arrangement of this is $5 + 12 = 5 + 3 \times 4$ and hence Paul takes 12 shots in his second session. Since he hits with $75\% = \frac{3}{4}$, of these shots, he hits the target 9 times in his second session.

19. Anurag leaves for school at 8:00 a.m. His school is 1 km away. When he walks, his speed is 4 km/h. When he cycles, his speed is 15 km/h. He is 5 minutes early when he walks.
How many minutes early is he when he cycles?

A 12 B 13 C 14 D 15 E 16

SOLUTION **E**

When Anurag walks the 1 km to school at 4 km/h, his journey takes him $\frac{1}{4}$ of an hour, which equals 15 minutes. Since he leaves at 8:00 a.m., he arrives at 8:15 a.m. and, since we are also told he arrives 5 minutes early, we can conclude that school starts at 8:20 a.m. When Anurag cycles the 1 km to school at 15 km/h, his journey takes him $\frac{1}{15}$ of an hour, which equals 4 minutes. Therefore he will arrive at 8:04 a.m. and will be 16 minutes early for the 8:20 a.m. start.

20. Jaina is a train driver. She drives the train from Sao Paolo to Rio. The journey usually takes 4 hours, non-stop and at constant speed. One day, the train stopped half way for 40 minutes. By what percentage did Jaina then need to increase the train's speed to still arrive on time?

A 30 B 40 C 50 D 60 E 70

SOLUTION **C**

To arrive on time, Jaina's train needs to travel the same distance in 40 minutes less than 2 hours, which is 80 minutes, as it would usually travel in 2 hours, or 120 minutes. Let the original speed of the train in km/minute be v and let the speed required be w. We know that distance = speed $\times$ time and hence $v \times 120 = w \times 80$. Therefore $w = \frac{120}{80} \times v$. Hence the new speed is $w = \frac{3}{2} \times v$ and hence w is 150 % of the original speed. Therefore Jaina needed to increase the train's speed by 50% to still arrive on time.

21. The letters p, q, r, s and t represent five consecutive positive integers, though not necessarily in that order. The sum of p and q is 69 and the sum of s and t is 72. What is the value of r?

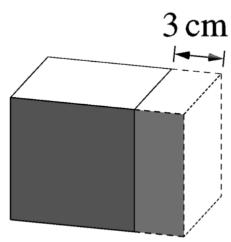
A 29 B 31 C 34 D 37 E 39

SOLUTION **C**

Since the integers are consecutive, the largest possible difference between them is four. The possible pairs of integers differing by four or less that add to 69 are 34, 35 and 33, 36. Similarly, the possible pairs of integers differing by four or less that add to 72 are 35, 37 and 34, 38.

If the pair adding to 69 were 34 and 35, then neither of the pairs adding to 72 could be made without repeating an integer. Therefore the pair adding to 69 is 33 and 36. Now the pair adding to 72 is 35 and 37, or we would have a difference greater than four between two of the integers. Therefore, four of the integers are 33, 35, 36 and 37 and hence the value of the missing integer, r, is 34.

22. When the length of a cuboid is reduced by 3 cm, its surface area is reduced by 60 cm^2. The resulting shape is a cube. What is the volume of the original cuboid, in cm^3?

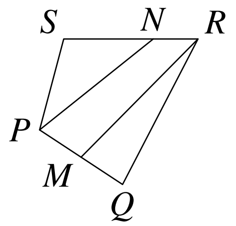


A 75 B 125 C 150 D 200 E 225

SOLUTION **D**

Since the resulting shape when the length of the cuboid is reduced by 3 cm is a cube, the right-hand face of the cuboid is a square. Let the side-length of this square be y cm. The reduction in surface area comes from four rectangles, each of dimensions 3 cm by y cm. Therefore $4 \times 3 \times y = 60$ and hence $y = 5$. Therefore the volume of the original cuboid, in cm^3, is $5 \times 5 \times (5 + 3) = 200$.

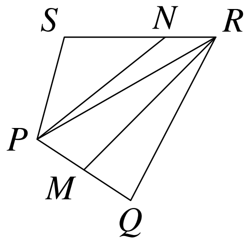
23. In the quadrilateral $PQRS$, the points M and N are marked on sides PQ and RS respectively so that $PM = MQ$ and $SN = 2NR$. The area of triangle MQR is 2 m^2, and the area of triangle PSN is 6 m^2.
What is the area of quadrilateral $PQRS$?



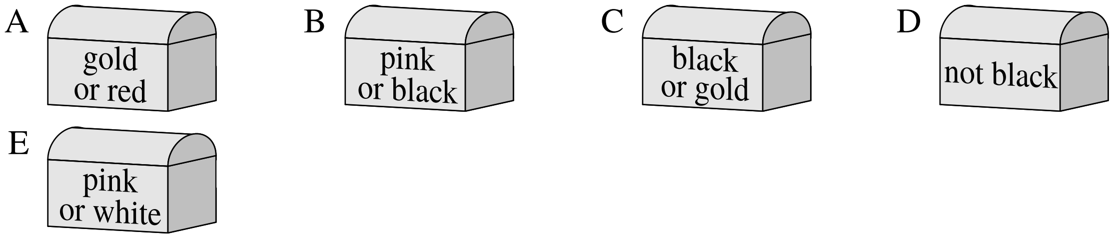
A 13 m^2 B 14 m^2 C 15 m^2 D 16 m^2 E 17 m^2

Solution **A**

Draw in line PR, as shown. Since $PM = MQ$, triangle PMR has both the same height as triangle MQR and an equal base. Therefore the area of triangle PMR is equal to the area of triangle MQR. Since $SN = 2 \times NR$, triangle PRN has half the base of triangle PSN and the same height. Therefore the area of triangle PRN is half the area of triangle PSN. Therefore the total area of quadrilateral $PQRS$, in m^2, is $2 + 2 + 6 + \frac{1}{2} \times 6 = 13$.



24. Adira keeps gold, red, black, pink and white pearls in five small boxes. Each box contains pearls of only one colour. The boxes are labelled as shown, and all the statements are true. Adira's friend Lilly wants to know which box contains the gold pearls. She may open exactly one of the five boxes to look inside.
Which box must Lilly open to be certain which of the boxes contains the gold pearls?



SOLUTION **D**

The answer is that Lilly should open box D. If box D contains the gold pearls, then she has found them. If box D contains red pearls, then she knows that box A, which we are told contains either red or gold pearls, contains the gold pearls. If box D contains pink pearls, then box B contains black pearls and she then knows that box C, which we are told contains black or gold pearls, contains the gold pearls. Finally, if box D contains white pearls, then box E contains pink pearls, box B contains black pearls and again she knows that box C contains the gold pearls.

It is possible to show that choosing any of the other boxes does not guarantee that Adira knows where the gold pearls are. For example, while opening box A could show her that the gold pearls were in box A, it could also show her that the red pearls are in box A and then the other boxes, in order, could contain pink, black, gold and white pearls *or* black, gold, pink and white pearls *or* black, gold, white and pink pearls. Hence she would not know whether the gold pearls were in box D or box C.

Showing that opening any one of boxes B, C or E also does not give sufficient evidence to find the gold pearls is left as an exercise for the reader.

25. Some birds, including Ha, Long, Nha and Trang, are perching on four parallel wires which are one above the other. There are 10 birds perched above Ha. There are 25 birds perched above Long. There are five birds perched below Nha. There are two birds perched below Trang. The number of birds perched above Trang is a multiple of the number of birds perched below her.
How many birds in total are perched on the four wires?

A 27 B 30 C 32 D 37 E 40

SOLUTION **A**

We shall prove that Long and just one other bird are perched on the lowest wire and hence that the total number of birds is 25 + 2 = 27.

For convenience, we label the first six sentences of the question as S1, S2, S3, S4, S5 and S6 in order and the four parallel wires as W, X, Y and Z from the top.

From S2 and S3, neither Ha nor Long can be perched on wire W and Long is perched on a wire below Ha.

From S4 and S5, neither Nha nor Trang can be perched on wire Z and Nha is perched on a wire above Trang.

Suppose that Nha is perched on wire W. Then, since Nha would then be above Ha, there would then be at most 9 other birds on wire W with Nha by S2. Hence, using S4, there would then be at most 9 + 1 + 5 = 15 birds in total, which contradicts S3. Therefore Nha is not perched on wire W and so is perched on wire X with Trang on wire Y.

Since Ha is not on wire W and Long is below Ha, Long is perched on either wire Y or wire Z. If Long were perched on wire Y, the same as Trang, there would be 25 birds above Trang and two birds below Trang, which contradicts S6. Therefore Long is perched on wire Z, which contains only two birds by S5.

Therefore the total number of birds is 25 + 2 = 27, as stated initially.

Note: While the argument above provides the solution to the question, it is also now possible to work out the exact number of birds perched on each wire. From the bottom, there are 2 birds, including Long, perched on wire Z, 3 birds, including Trang, perched on wire Y, 12 birds including Ha and Nha perched on wire X and 10 birds perched on wire W.